121B: ALGEBRAIC TOPOLOGY

Outline of course

Math 121b will cover basic topics in cohomology and homotopy theory. Given any topological space X and associative ring R there is a graded associative ring $H^*(X;R)$ which is a contravariant homotopy functor of X in the sense that given any continuous map $f : X \to Y$ we get an induced graded ring map $f^* : H^*(Y;R) \to H^*(X;R)$ which depends only on the homotopy class of f. We will spend a lot of time studying the duality between homology and cohomology for manifolds of various kinds (closed, compact with boundary and noncompact).

We will also cover basic topics in homotopy theory such as homotopy excision, which says that homotopy groups $\pi_n(X)$ satisfy excision in a certain range of dimensions, and the Hurewicz Theorem which says that the first nonzero homotopy group, if abelian, is isomorphic to the first nonzero homology group of a space X.

We will follow the second half (Chapters 2 and 3) of Allen Hatcher's book "Algebraic Topology." The grading will be based on weekly homework and in-class discussions. Students need to participate in class discussions. The things they say in class do not need to be correct, just thoughtful.

The topics that will be covered in class, on a week-by-week basis are the following.

(0) (Jan 18) Examples of spaces, Δ-complexes and cocycles.
(1) (Jan 23) Universal Coefficient Theorem (UCT) and cohomology of spaces.
(2) (Jan 30) Cup product and the cohomology ring $H^*(X;R)$.
(3) (Feb 6) Künneth Formula, Eilenberg-Zilber Theorem, spaces with polynomial cohomology.
(4) (Feb 13) Poincaré Duality and orientation of manifolds.
(5) (Feb 27) More duality on manifolds.
(6) (Mar 6) Duality on noncompact manifolds.
(7) (Mar 13) Alexander Duality.
(8) (Mar 20) CW complexes and homotopy groups.
(9) (Mar 27) Whitehead Theorem, cellular approximation.
(10) (Apr 3) CW approximation.
(11) (Apr 10) Homotopy excision.
(12) (Apr 24) Hurewicz Theorem.
(13) (May 1) Discussion of additional topics.