

Due Wednesday, September 12. Final revised version due Sept 26.

Be sure to write clearly, using complete sentences. Do not use symbols such as \forall , \exists , \rightarrow , \Rightarrow , $\Rightarrow\Leftarrow$, \therefore , $\sqrt{\quad}$, etc., and do not use abbreviations like s.t., w/, w/o, etc. In all problems you must prove that your answer is correct, even if the problem does not explicitly ask you to do so (as in problems 2 and 4).

1. Let $A = \{k^2 + k \mid k \in \mathbb{Z}\}$ and let $B = \{n^2 - n \mid n \in \mathbb{N}\}$. Prove that $A = B$.
2. Determine the set of real solutions to the inequality

$$\left| \frac{x}{x-2} \right| \leq 1.$$

[Hint: Prove that your answer A is the same set as the set of solutions S .]

3. Let x and y be real numbers. Prove that $(x + 3y)^2 \leq (1 + x^2)(1 + 9y^2)$. [Hint: The statement $a \leq b$ is equivalent to the statement $b - a \geq 0$.]
4. Determine the image of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

$$f(x) = \frac{x^2}{2 + x^2}.$$

In other words, determine which real numbers that can be expressed in the form $x^2/(2+x^2)$ for some x . [Don't forget to prove that your answer is correct.]