

Due Wednesday, September 19. Final revised version due October 8.

Be sure to write clearly, using complete sentences. Do not use symbols such as \forall , \exists , \rightarrow , \Rightarrow , $\Rightarrow\Leftarrow$, \therefore , $\sqrt{\quad}$, etc., and do not use abbreviations like s.t., w/, w/o, etc. In all problems you must prove that your answer is correct, even if the problem does not explicitly ask you to do so (as in problems 2 and 4).

1. Let $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Q}$ be given by $f(a, b) = (a - 3)(a + 2b)/2$.
 - a) Show that the image of f is contained in \mathbb{Z} . What is the appropriate interpretation of the words “contained in” in this context?
 - b) Is the image of f equal to \mathbb{Z} ? Prove it or give a counterexample.

2. Suppose that f, g are two real valued functions on \mathbb{R} . Determine which of the following statements is true. Prove the statement if it is true. Find a counterexample and explain why it is a counterexample if it is false.
 - a) If f is increasing and g is nonincreasing then $f - g$ is increasing.
 - b) If f, g are both unbounded then the product fg is unbounded.
 - c) If $f + g$ and $f - g$ are both bounded then f, g are both bounded.

3. Let $f : A \rightarrow B$ be a mapping. For every subset S of A let $dh(S)$ be the set of all $b \in B$ so that there are at least two elements of S which map to b (i.e., $b = f(s_1) = f(s_2)$ is “double hit”). Let X, Y be two subsets of A .
 - a) Does $dh(X \cup Y)$ contain $dh(X) \cup dh(Y)$?
 - b) Does $dh(X \cap Y)$ contain $dh(X) \cap dh(Y)$?What is the appropriate interpretation of the word “contain” in this context?