Due Wednesday, Sept 26. (Final version of HW1 due Monday, Oct 1.)

From the textbook, chapter 2:

2.4

2.21

2.22

2.44

Note that Appendix B of the textbook has hints for some problems. (In math, "some" means "one or more".)

Also do the following problems:

1. Consider the following statement:

   For all positive real numbers x and y, if $x + y \geq 20$ then $x \geq 11$ or $y > 5$.

   Prove this in two ways:

   (a) By proving the contrapositive.

   (b) With a direct proof by cases. (Hint: Take one case to be $x \geq 11$. What should the other case be?)

2. (a) Express the following statement using logical connectives, quantifiers, and the operations of arithmetic

   The only numbers which are equal to their squares are 0 and 1.

   (b) Suppose that f is a function from $\mathbb{R}$ to $\mathbb{R}$. Using logical connectives and quantifiers express the statement that the image of f is the set $\{\, y \mid y > 2 \,\}$. For example, your statement might look something like this (though this isn't correct):

   $$\forall x \, \exists y \big(f(x) = y \leftrightarrow y > 2 \big)$$

   In both parts you may assume that all variables are quantified over the set of real numbers. (So you may write, for example, $\forall x$ rather than $\forall x \in \mathbb{R}$.)