Math 23b	Homework 6	Fall, 2007

Due October 25.

1. Let f be a function from $\mathbb{R}$ to $\mathbb{R}$. (Recall that $\mathbb{R}$ is the set of real numbers.) Prove that f cannot be both bounded and surjective.

2. A function $g : A \to A$ is called *idempotent* if it has the property that $g(g(x)) = g(x)$ for all $x \in A$. Prove that if $g : A \to A$ is 1-1 and idempotent then g is the identity function on A.

3. Let $A_1, A_2, \ldots$ be sets, each of which is finite or countable. Prove the the union of all of these sets is countable. [First prove it in the case when the sets are pairwise disjoint.]

4. Recall that if A is a set, its power set $\mathcal{P}(A)$ is the set of all subsets of A. Find an injection from $\mathcal{P}(\mathbb{N})$ to $\mathbb{R}$.