Math 23b	**Answers to Quiz 1**	**Oct, 2007**

1. [5 points] Draw a truth table for the statement $(\neg p \vee q) \vee (p \Rightarrow q)$. Is it a tautology?

p	q	$\neg p$	$\neg p \vee q$	$p \Rightarrow q$	$(\neg p \vee q) \vee (p \Rightarrow q)$
T	T	F	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

This is not a tautology since it is false in the second case.

2. [8 points] Find a counterexample to the following statement and explain why it is a counterexample. [Use words! Make complete sentences!]

If f and g are functions from $\mathbb{R}$ to $\mathbb{R}$ with the property that either f is bounded and nondecreasing or g is bounded and nondecreasing then $f + g$ is either bounded or nondecreasing.

[Hint: Constant functions (e.g. $f(x) = 1$) are bounded and nonincreasing.]

Let $f(x) = 0$ and $g(x) = -x$. Then f is bounded and nondecreasing so the hypothesis is true, but $h(x) = f(x) + g(x) = -x$ is neither bounded nor nondecreasing. It is not bounded since $|f(x) + g(x)| = x$ goes to ∞ as x goes to ∞ and it is not nondecreasing since $h(1) = -1 < h(0) = 0$.

3. [10 points] a) Write the following statement in logical notations,

The image of the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$ is contained in the set of positive real numbers.

The statement "The image of $f : \mathbb{R} \to \mathbb{R}$ is contained in S" can be written: $(\forall x \in \mathbb{R}) f(x) \in S$. So, the answer is:

$$(\forall x \in \mathbb{R}) x^2 > 0$$

b) find its negation

$$(\exists x \in \mathbb{R}) x^2 \leq 0$$

c) write the negation in English without using words of negations (such as "not" or "no" or "non-negative" or "non-positive" or $\not\leq$).

There is a real number whose square is less than or equal to zero.

d) Then prove the negation.

Proof: The negation holds since 0 is a real number whose square is ≤ 0.

4. [7 points] If a, b are positive real number so that $2a - b \leq 9$ then prove, using the contrapositive, that either $a \leq 12$ or $b > 15$. [Don't forget to write in complete sentences.]

The contrapositive statement is:

If a, b are positive real numbers so that $a > 12$ and $b \leq 15$ then $2a - b > 9$.

To prove this suppose that $a > 12$ and $b \leq 15$. Then $2a > 24$ and $b + 9 \leq 15 + 9 = 24$ Therefore,

$$2a > 24 \geq b + 9$$

which implies that

$$2a > b + 9$$

Subtracting b from both sides gives

$$2a - b > 9$$

Therefore, the contrapositive statement holds. So, the original statement also holds.