

Rules: Closed book. No calculators. Write all answers in your exam booklet. Write clearly, using complete sentences.

1. [10 points] Find a counterexample to the following statement.

If f and g are functions from \mathbb{R} to \mathbb{R} with the property that $f + g$ is bounded and $f(x) \geq 0$ for all x in \mathbb{R} then g is bounded.

2. [10 points] Write the following statement in logical notation, find its negation, then write the negation in English. Use P for the set of all primes and O for the set of odd integers.

For every positive integer n , if $n + 7$ is a sum of two primes then n is odd.

3. [10 points] Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by the formula $f(x) = |x| + |1 - x|$. Prove that the minimum value of f is 1.