

Rules: Closed book. No calculators. Write all answers in your exam booklet. Write clearly, using complete sentences.

1. [10 points] Find a counterexample to the following statement.

If $f : A \rightarrow B$ is a function from set A to set B and C, D are subsets of A so that $f(C) = f(D)$ then $C = D$.

2. [5 points] Suppose that you are going to prove the following statement by contradiction. How would the proof start? (You do not have to finish the proof!)

If $|a| < 4$ then for all $x \in \mathbb{R}$, $x^2 + ax + 4 \geq 0$.

(In this statement, a is a real number.)

3. [5 points] If x, y are positive real numbers so that $xy \geq 20$ then prove, using the contrapositive, that either $x \geq 4$ or $y > 5$.

4. [10 points] Find the image of the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \frac{2}{1+x^2}$$

and prove it.