5.10. **Homework 6.** Due Thursday, March 18. The first 5 problems are from the worksheets. Look there for detailed instructions and hints.

(1) Write down an explicit 1-1 correspondence between the subsets of $[n]$ and functions $g : [n] \to \{0, 1\}$.

(2) Show that the union of two countable sets is countable.

(3) If $g \circ f$ is a bijection, then show that g is onto and f is 1-1.

(4) Find a bijection between the sets $\mathbb{R}$ and $(-\infty, 0] \cup [1, \infty)$ and prove that your mapping is a bijection. [Hint: Write $\mathbb{R}$ as a union $\mathbb{R} = (-\infty, 0] \cup (0, \infty)$. The easiest method to prove your map is a bijection is to find the inverse mapping g and show $f \circ g, g \circ f$ are both identity mappings on their respective domains.]

(5) Find a bijection between the set of all functions $[n] \to [3]$ and the set of all integers from 1 to 3^n and prove that your mapping is a bijection. [You may assume without proof that the set B^A of all mappings $f : A \to B$ has exactly m^n elements if $|A| = n$ and $|B| = m$ are finite. (with the indeterminate $0^0 = 1$). So, the pigeonhole principle applies.]

(6) Find an example of three sets A, B, C and three mappings $f : A \to B, g : B \to C, h : C \to A$ so that the composition

$$h \circ g \circ f : A \to A$$

is the identity mapping on A but

$$g \circ f \circ h : C \to C$$

is not the identity mapping on C.

Make sure that you give specific sets A, B, C and functions f, g, h. These can be given by formulas or by a list of values (when the sets are finite). Show that the functions have the required properties by doing the appropriate calculations.

(7) Write in words the following two statements. Here $(\exists S)$ means "there exists a set S"

$$(\exists S)(\forall T)(\forall f \in T^S)(\exists g \in S^T)(\forall x \in S)g(f(x)) = x$$
$$(\forall S)(\exists T)(\forall f \in T^S)(\exists g \in S^T)(\forall x \in S)g(f(x)) = x$$

Give one example to show that the second statement is not true. (You don't have to prove the first statement.)

(8) Write the following statement in logical notation (similar to problem (7)). For any surjective mapping $f : S \to T$ between any two sets there is a mapping $g : T \to S$ so that $f \circ g : T \to T$ is the identity mapping on T. Then write its negation in symbols and in words. [This is one form of the Axiom of Choice. It is impossible to prove or disprove this statement. So, don't try!]