0. Quiz 0

Rules: Closed book. Notes on one sheet of letter sized paper allowed. (Prepare one for the next quiz!)

(1) What is the statement of the triangle inequality? Give an example.

The triangle inequality says: For all real numbers x, y we have

$$|x+y| \leq |x| + |y|$$

for example, if $x = 5$ and $y = -2$ then $|x + y| = |5 - 2| = 3$ is $\leq |x| + |y| = 2 + 3 = 5$.

(2) Find the solution set of the inequality $x^2 + 4x + 4 \leq 9$. (No proof needed.)

The solution set is the closed interval $[-5, 1]$.

The explanation (which was not required) is that $(x + 2)^2 \leq 9$ gives $|x + 2| \leq 3$ which is the same as $-3 \leq x + 2 \leq 3$. Subtract 2 from all three terms to get the answer. (This explanation proves $S \subseteq [-5, 1]$.)

(3a) What is the definition of a bounded function?

(3b) What is the definition of an increasing function?

(3c) Find an example of an increasing function $f : \mathbb{R} \to \mathbb{R}$ which is bounded.

A *bounded function* is a function $f : A \to \mathbb{R}$ with the property that there is a positive real number M so that $|f(a)| \leq M$ for all $a \in A$. An *increasing function* is function $f : B \to \mathbb{R}$ whose domain is a subset $B \subset \mathbb{R}$ so that $f(a) < f(b)$ whenever a, b are two elements of B so that $a < b$.

An example of an increasing bounded function if $f(x) = x$ on the domain $B = [0, 1]$. This is bounded by $M = 1$ since $|x| \leq 1$ for all x in the interval $[0, 1]$. The function is increasing since $a < b$ implies $f(a) = a < b = f(b)$.

(4) Prove, by contradiction, that if $A - B = A$ then $A \cap B$ is the empty set.

Suppose by contradiction that $A \cap B$ is nonempty. Then it has an element, say $a \in A \cap B$. Then $a \in A$ and $a \in B$ which implies that $a \notin A - B$ since the set $A - B$ is the set of all $x \in A$ so that $x \notin B$. Therefore, A is not contained in the set $A - B$ which contradicts the assumption that $A = A - B$. This proves that $A \cap B$ cannot be nonempty. So, it must be empty as claimed.