

5.8. **Review 2a.** This is the first review sheet of Quiz 2. The topics for Quiz 2 are induction and strong induction and bijections and cardinality. Countability will not be on Quiz 2 but will be on HW 6.

(1) Calculate the sum

$$\sum_{k=1}^n \frac{3k-2}{5}$$

(2) Prove that

$$\sum_{k=1}^n 3^k = \frac{3^{n+1} - 1}{2}$$

(3) If $f : A \rightarrow B$ is surjective but not injective and $g : B \rightarrow C$ is injective but not surjective then does it follow that $g \circ f$ is a bijection? Does it follow that $g \circ f$ is not a bijection?

(4) Given a function $f : A \rightarrow B$, write in logical notation the statement that f is injective but not surjective.

(5) Find a function $f : [0, 1) \rightarrow \mathbb{R}$ which is surjective but not injective.

(6) You have three kinds of coins that you use as weights. They weight 3gm, 5gm, 6gm. Show that you can use your coins on one side of a scale to balance any integer number of weights ≥ 8 on the other side of the scale using strong induction.

(7) Given an example of a function $f : A \rightarrow A$ which is surjective but not 1-1.