

4.8. **Worksheet.**

4.8.1. Draw a truth table for the statement  $(p \vee \neg q) \wedge (p \Rightarrow r)$ . Is this a tautology?

If  $A, B, C$  are sets and  $P(x), Q(x), R(x)$  are the assertions  $x \in A, x \in B, x \in C$  respectively, then write the set

$$D = \{x \in U \mid (P(x) \vee \neg Q(x)) \wedge (P(x) \Rightarrow R(x))\}$$

in terms of union, intersection, complements, etc using  $A, B, C$ . [Hint: Look at your truth table.]

4.8.2. Is the following function bounded? increasing 1-1?

$$f : (-\infty, 0] \cup [1, \infty) \rightarrow \mathbb{R}$$

given by

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x \geq 1 \\ 2 - 3x & \text{otherwise} \end{cases}$$

Find a real valued function on the half open interval  $(0, 1]$  which is not bounded, not increasing and not 1-1.

4.8.3. Find for which positive integers  $n$  the following inequality holds and prove it by induction.

$$n^2 > 5n - 4$$

4.8.4. Write the following statement in logical notation, find its negation and write the negation without using words of negation such as “not” or “no” or “without” or “nonnegative” or “nonpositive” or any symbol with a line through it ( $\neq, \not\leq$ , etc)

*The image of the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = x^2$  is contained in the set of positive real numbers.*

4.8.5. If  $a, b$  are positive real numbers so that  $2a - b \leq 9$  then prove that either  $a \leq 12$  or  $b > 15$ . Write your answer in complete sentences.