1 Induction

[Here is an example of mathematical induction that I did in class. Homework is at the bottom of the page.]

Definition 1.1. $C^1(\mathbb{R})$ is the set of continuously differentiable functions on $\mathbb{R}$. This is a vector space over $\mathbb{R}$. A *derivation* on $C^1(\mathbb{R})$ is defined to be a linear mapping

$$D : C^1(\mathbb{R}) \to C^1(\mathbb{R})$$

satisfying the condition

$$D(f \cdot g) = (Df)g + fDg$$

for all $f, g \in C^1(\mathbb{R})$. Here $f \cdot g$ is the product, not the composition.

Theorem 1.2.

$$D(x^n) = nx^{n-1}Dx \tag{1}$$

for all integers $n \geq 1$.

[Note that the letter x is always real and n is always a nonnegative integer unless otherwise stated.]

Proof. The proof is by induction on n. If $n = 1$ the statement is obvious. [Other wording are: "the statement is a tautology." or "there is nothing to prove."] Assume by induction that $n \geq 1$ and statement holds for n. Then

$$D(x^{n+1}) = D(x^n x) = D(x^n)x + x^n Dx$$

since D is a derivation. By the induction hypothesis this is equal to

$$nx^{n-1}(Dx)x + x^n Dx = [nx^n + x^n]Dx = (n + 1)x^n Dx.$$

This is Equation (1) in the case $n+1$. Thus the formula holds for all integers $n \geq 1$. □

[It is difficult to know how much you can say. You have to have words but not too many.]

Homework (due Monday, 9/9/02)
#1.1. Figure out the formula for the sum of the first n odd numbers and prove it by induction.

#1.2. Find a formula for $1 + \sum_{j=1}^{n} j!j$ and prove it by induction.

#1.6. Prove [by *induction*] that the sum of the first n squares is given by whatever it is.

#1.13. Prove [by induction on $n \geq 1$] that $(1 + x)^n \geq 1 + nx$ if $1 + x > 0$. [Assume that x is real.]

#1.18. Prove that there are n consecutive composite numbers for all $n \geq 2$. [By induction!!]