

## 7 About Homework 7:

1) There are several ways to show that every group of order 4 is abelian. Let

$$G = \{e, a, b, c\}.$$

The elements  $a, b, c$  must have order 2 or 4. There are two possibilities.

1.  $a, b, c$  all have order 2.
2. At least one of them, say  $a$ , has order 4.

In the second case,  $G$  is cyclic ( $G = \langle a \rangle$ ) and is therefore abelian. In the first case, every element is its own inverse so

$$gh = (gh)^{-1} = h^{-1}g^{-1} = hg$$

for any  $g, h \in G$ . (This is the trick of changing a formula into words and then back into a formula using different symbols.)

Another way: Take any two elements  $g, h \in G$ . We have to show that they *commute* ( $gh = hg$ ).

1. If either  $g$  or  $h$  is  $e$  then  $gh = hg$  so suppose that they are not  $e$ .
2. If  $g = h$  then  $gh = g^2 = hg$  so suppose  $g \neq h$ .
3. If  $gh = e$  then  $h = g^{-1}$  so  $hg = e = gh$  so suppose  $g, h$  are not inverse to each other. Then  $gh$  must be the third nonidentity element of the group and  $hg$  must also be the third nonidentity element of the group so  $gh = hg$ .

If you used the multiplication table, the table must be symmetric (equal to its transpose) in order for the group to be abelian.

2) Two nonisomorphic groups of order 6 are  $\mathbb{Z}/6$  and  $S_3$ . These are nonisomorphic since  $\mathbb{Z}/6$  is abelian and  $S_3$  is not. (When two groups are isomorphic they have the same intrinsic properties: They have the same order, the same number of elements of any given order, the same number of conjugacy classes. If one is abelian so is the other. If one is cyclic, so is the other.) The book also has this on page 180.