17 Hints on last Homework:

5.17 Show that every element of order 2 in A_5 commutes with only two other elements of order 2 in A_5. Conclude that the Sylow 2-subgroups are disjoint.

5.18 Do 312, 1000 first because they are easy. For $n = 300$ look at the Sylow 5 subgroups. In the case when there are exactly 6 Sylow 5 subgroups, we get a homomorphism $\phi : G \to A_6$ as in the proof of Lemma 5.20. For $n = 616$ show that any simple group of order 616 must have 560 elements of order 11 and 132 elements of order 7.

5.23 The *commutator subgroup* G' is a normal subgroup of G having the property that G/G' is abelian. (See Proposition 5.38).

5.26 (i) If $p < q$ then show that G has a normal Sylow q-subgroup.

5.26 (ii) Break into cases. Case 1: $p < q$, Case 2: $p > q$.