

1 About Quiz 1:

Question 2: Consider the following statement:

“Any positive integer n can be written in the form $n = 3a + 5b$ where a, b are positive integers.”

I wrote this in a deliberately ambiguous language because the question was to write it in symbols which are, by nature, necessarily precise.

The symbolic representation of this statement is:

$$(\forall n \in \mathbb{Z}_+)(\exists a, b \in \mathbb{Z}_+) n = 3a + 5b.$$

The quantifier should always be at the beginning. There are only two possible quantifiers: \forall, \exists . It is standard to put parentheses around the quantifier clauses but commas are also acceptable. There is an implied “such that” at the end of each existential clause which is usually omitted in the notation when parentheses are used. Note that every variable in the statement needs to be quantified.

In general, the word “where” is an existential quantifier (\exists) and “when” is a universal quantifier (\forall). However, the word “where” is ambiguous. For example, does

$$S \text{ contains } 2n \text{ where } n \in \mathbb{Z}$$

mean the set S contains all even integers or does it mean that S contains at least one even integer?

A correct answer to the first question is:

Definition 1.1. An *odd integer* is an integer n so that $n \neq 2a$ for any integer a .

Most students wrote the following:

An *odd integer* is an integer n , $n = 2a + 1$ where a is an integer.

I corrected this to:

An *odd integer* is an integer n which can be written in the form $n = 2a + 1$ where a is an integer.

But, it is definitely more precise to use the clause “for some integer a ” in place of “where a is an integer.”