

## 2 About Quiz 2:

The first question was:

How many zeroes are at the end of the number 100!? Noone got this correct. The answer was 24. The formula explained in class was

**Theorem 2.1.** *If  $p$  is prime then the number of times that  $p$  divides  $n!$  is equal to*

$$\left[ \frac{n}{p} \right] + \left[ \frac{n}{p^2} \right] + \left[ \frac{n}{p^3} \right] + \dots$$

*Proof.* Dividing  $n$  by  $p$  gives

$$n = pq + r$$

where  $0 \leq r < p$ . Then the only factors of  $n!$  which are divisible by  $p$  are

$$p, 2p, 3p, \dots, qp.$$

The product of these numbers is  $p^q$  times  $q!$ . By induction the number of times that  $p$  divides this product is

$$q + \left[ \frac{q}{p} \right] + \left[ \frac{q}{p^2} \right] + \left[ \frac{q}{p^3} \right] + \dots$$

But  $q = [n/p]$ ,  $[q/p] = [n/p^2]$ , etc., proving the theorem.  $\square$

Almost everyone got the induction question correct but did not have enough words in the proof.

The last words should be something like: "Since the product of two odd numbers is odd, the answer has the required form for  $n + 1$ . Therefore the equation holds for all  $n \geq 1$ ."