MATH 30A: QUIZZES

2. Review for Quiz 2

Quiz 2 is on Thursday, Oct 5. It will have 4 questions. Open book and notes. You may also bring a calculator and/or laptop computer. But no fair using internet or IM with friends!

2.1. **isomorphisms.**

2.1.1. Find an example of two infinite groups with the same number of elements which are not isomorphic and give a reason.

2.1.2. Write down an explicit isomorphism between the groups $G = (\mathbb{R}, +)$ and $H =$ the subgroup of $GL(2, \mathbb{R})$ given by

$$H = \left\{ \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix} \mid x \in \mathbb{R} \right\}$$

2.1.3. Find an example of an automorphism ϕ of a nontrivial group G so that $\phi(x) \neq x$ for all nontrivial ($x \neq e$) elements of G.

2.1.4. Find as many (nonisomorphic) examples as you can of groups G so that $\mathrm{Aut}(G)$ has only one element.

2.2. **permutations.**

2.2.1. Find an element of S_{12} of order 60.

2.2.2. Show that all permutations of odd order are even.

2.2.3. Prove the following formula:

$$\sigma(123 \cdots n)\sigma^{-1} = (\sigma(1)\sigma(2)\sigma(3) \cdots \sigma(n))$$

2.3. **cyclic groups.**

2.3.1. Find as many (nonisomorphic) examples as you can of cyclic groups having only two generators.

2.3.2. Suppose that a, b are elements of a group G and $ab = ba$. Suppose that $|a| = 8, |b| = 30$. What are the possible values of $|ab|$?

2.3.3. Show that aba^{-1} has the same order as b.

Date: October 1, 2006.

2.4. **subgroups.**

2.4.1. How many subgroups does D_4 have? List them. (You don't have to list the elements of each.)

2.4.2. Suppose that H, K are subgroups of a group G which centralize each other in the sense that $hk = kh$ for all $h \in H, k \in K$. Then show that the set

$$HK := \{hk \,|\, h \in H, k \in K\}$$

is a subgroup of G.

2.4.3. The group $\mathbb{Z}_8$ has eight elements and only one subgroup of order 4. The group D_4 has order 8 and 3 subgroups of order 4 (check your answer to 2.4.1). Can you find a group of order 8 which has more than 3 subgroups of order 4? [Hint: look at Practice quiz 1.4 where X has 3 elements.]