

MATH 30A: QUIZZES

3. ANSWERS TO QUIZ 3A

Here was the question: Take the group $G = S_3 \oplus \mathbb{Z}_4$ where S_3 is the symmetric group on 3 letters.

- (1) What are the possible orders of the elements of G ?
Since $|G| = 6 \cdot 4 = 24$, the possible orders of the elements are 1, 2, 3, 4, 6, 8, 12, 24
Find an element of each order if you can.
The actual orders that you get are 1, 2, 3, 4, 6, 12. (There is no element of order 8 or 24). Elements of these orders are:

$$(e, 0), ((12), 0), ((123), 0), (e, 1), ((123), 2), ((123), 1)$$

- (2) Let H be the cyclic subgroup generated by the element $((12), 1)$ (The first coordinate is the transposition $(12) \in S_3$ and the second is the generator $1 \in \mathbb{Z}_4$.)

$$H = \{(12), 1), (e, 2), ((12), 3), (e, 0)\}$$

What is the index of H in G ?

Since $((12), 1)$ has order 4, the index is $|G : H| = |G|/|H| = 24/4 = 6$.

Find the left cosets of H .

There are 6 left cosets:

$$H, ((123), 0)H, ((132), 0)H, (e, 1)H, ((123), 1)H, ((132), 1)H$$

- (3) Show that $((13), 3)$ and $((123), 1)$ do not lie in the same left coset of H in G .

Let $a = ((13), 3)$ and $b = ((123), 1)$. Then

$$a^{-1}b = ((13), 1)((123), 1) = ((13)(123), 2) = ((12), 2)$$

This is not one of the 4 elements of H . So, $aH \neq bH$ and a, b do not lie in the same left coset of H .