MATH 30A: QUIZZES

3. Review for Quiz 3b

Here are the answers to the review questions that we discussed in class and some new problems.

3.1. orders of elements in G/N.

3.1.1. Give an example of a normal subgroup N of a group G and an element $a \in G$ so that the order of a is not equal to the order of aN considered as an element of G/N.

The examples that we looked at were all from $G = D_4$ with $N = Z(D_4) = \{e, r^2\}$. First we took $a = r^2$. This has order 2 in $G = D_4$ but $r^2N = N$ has order 1 in the factor group G/N.

Next we took $a = r$. This has order 4 but rN has order 2.

3.1.2. What is the "formula" for the order of aN considered as an element of G/N?

The order of aN is equal to the smallest positive integer n so that $a^n \in N$. In the example, $r^2 \in N$ so rN has order 2.

3.1.3. *New problem.* Suppose N is a normal subgroup of a group G and $|N| = 9$, $|G| = 900$. If aN has order 4 in G/N, what are the possible values of $|a|$?

3.2. subsets of factor groups.

3.2.1. Give an example of the theorem:

Theorem 3.1. *Suppose that N is a normal subgroup of a group G and H is a subgroup of G which contains N. Then:*

1. *N is a normal subgroup of H and*
2. *H/N is a subgroup of G/N.*

The example that we looked at was again $G = D_4$, $N = Z(D_4) = \{e, r^2\}$ and $H = \langle r \rangle = \{e, r, r^2, r^3\}$

N is a normal subgroup of H (it has index 2).

H/N is a subgroup of G/N. The factor group G/N has four elements:

$$G/N = \{N, rN, sN, srN\}$$

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After some fumbling around we figured out that:

$$H/N = \{N, rN\}$$

Since rN has order 2 in G/N, it generates a subgroup of G/N with two elements. This subgroup is H/N.

Which group is G/N? Is it $\mathbb{Z}_4$ or $\mathbb{Z}_2 \oplus \mathbb{Z}_2$?

3.2.2. *New problem.* Find all subgroups of the factor group $\mathbb{Z}/4\mathbb{Z}$. Find a subset of $\mathbb{Z}/4\mathbb{Z}$ which is not a subgroup.

3.3. **Is this normal?**

3.3.1. Let $G = S_3 \oplus \mathbb{Z}_2$. Let $H = \langle((123), 0)\rangle$.

(1) Is H a normal subgroup of G?
(2) If not, explain.
(3) If so, what is the factor group G/H?

We decided that H is a normal subgroup of G and we proved it using the fact that $(a, b)^{-1} = (a^{-1}, -b)$ since the second coordinate is an additive group.

$$\begin{aligned}(a, b)((123), 0)(a^{-1}, -b) &= (a(123)a^{-1}, b + 0 - b)\\ &= ((123), 0) \text{ or } ((132), 0) = ((123), 0)^2\end{aligned}$$

Note: $a(123)a^{-1} = (a(1)a(2)a(3))$ is one of the two 3-cycles of S_3.

Since $|G| = 6 \times 2 = 12$ and $|H| = 3$, the factor group has 12/3=4 elements:

$$G/H = \{H, ((12), 0)H, (e, 1)H, ((12), 1)H\}$$

Since each nontrivial element has order 2, this is isomorphic to $\mathbb{Z}_2 \oplus \mathbb{Z}_2$.

3.3.2. *New problem.* Let $G = S_3 \oplus \mathbb{Z}_3$. Let $H = \langle((123), 1)\rangle$.

(1) Is H a normal subgroup of G?
(2) If not, explain.
(3) If so, what is the factor group G/H?

3.3.3. *Another problem.* If N is a normal subgroup of G and H is another normal subgroup of G containing N, then show that H/N is a normal subgroup of G/N.

What is $(G/N)/(H/N)$?