MATH 30A: QUIZZES

3. Review for Quiz 3

Quiz 3 is on Thursday, Oct 19. It will have 1 question. Open book and notes. You may also bring a calculator and/or laptop computer. But no internet or IM with friends. No cellphone use. The quiz will be on cosets and direct products.

3.1. **Cosets.**

3.1.1. List all the left cosets of $H = \langle r^2 \rangle$ in the group D_{2n} which consists of the rotations $e, r, r^2, \cdots, r^{2n-1}$ where $r = R_{\pi/n}$ and the reflections $s, sr, sr^2, \cdots, sr^{2n-1}$. Find all the right cosets.

3.1.2. Prove that Ha is the set of all inverses of elements of $a^{-1}H$.

3.1.3. Let $G = S_6$ (the symmetric group on 6 letters). $H = \langle (123) \rangle$. Find two elements $a, b \in S_6$ so that $aH = bH$ but $Ha \neq Hb$.

3.1.4. Suppose that H, K are subgroups of G and $H \subseteq K$. Then show that every left coset of H is contained in a left coset of K. State and prove the converse.

3.2. **Direct products.**

3.2.1. If $G \oplus H$ is abelian then prove that G and H are abelian.

3.2.2. Find all groups G, H so that $G \oplus H$ is cyclic.

3.2.3.

(1) If $a \in G, b \in H$ have order $|a| = 4, |b| = 6$, what is the order of (a, b) in the product group $G \oplus H$?
(2) (a, b) (as in (1)) generates a subgroup $\langle (a, b) \rangle$ of $G \oplus H$ which is not of the form $A \oplus B$. Why not?
(3) Complete the sentence: If $a \in G$ and $b \in H$ then the subgroup $\langle (a, b) \rangle$ of $G \oplus H$ is of the form $A \oplus B$ if ....

3.2.4. If H, aH, bH are the left cosets of H in G_1 and K, cK are the left cosets of K in G_2 what are the left cosets of $H \oplus K$ in $G_1 \oplus G_2$? What are the right cosets of $H \oplus K$ in $G_1 \oplus G_2$?

Date: October 18, 2006.