

5. REVIEW FOR QUIZ 4

Quiz 4 is on Thursday, Nov 16. It will have 2 questions. Open book and notes. You may also bring a calculator and/or laptop computer. But no internet or IM with friends. No cellphone use. The quiz will be on cosets and direct products.

5.1. First Isomorphism Theorem.

5.1.1. *statement.* What does the first isomorphism theorem say? Give an example using a homomorphism which is neither onto or 1-1. In particular, find such a homomorphism from D_4 to S_3 .

5.1.2. Suppose that G is a finite group and $\phi : G \rightarrow H$ is a homomorphism. Then show that $\phi(P)$ is a Sylow p -subgroup of $\phi(H)$ for any Sylow p -subgroup of G . (Dumb=too simple question.)

5.2. Sylow theorems.

5.2.1. If G is a group of order pq where $p < q$ then show that either the Sylow p -subgroup of G or the Sylow q -subgroup of G is normal. (Also dumb)

How many elements of order p are there?

5.2.2. If $|G| = pqr$ where $p < q < r$ are prime. Take $p = 5, q = 7, r = 11$, what can you say? Which Sylow subgroups are normal? How many are there when they are not normal?

5.2.3. Suppose that P is a normal Sylow p -subgroup of a finite group G . Then show that P contains all elements of G of order p .

5.2.4. Show that groups of order p^2q where $p < q$ are prime. Then show that G has a normal Sylow subgroup. (I'll send you hints later.)