1. MATH 30A, FALL 2009
HOMEWORK 1

Due: Thursday, Sept 3, at noon (in class).

Homework counts 50% of your grade. Late homework will incur a penalty.

State the questions and answer them in complete sentences or as you would speak. (Symbols are nouns and equations are also sentences. For example, "$a = b$ but $c \neq d$" is a sentence.)

1.1. **Problems from book.** Page 8, number 6, 10, 18, 29, 32, 36

1.2. **Additional questions.** These are intended to be more challenging but not impossible. It is better to write a precise and correct partial answer than a partially incorrect complete answer. Try to put the proper words around your equations.

a) Prove by induction on n that $3^n > n^3$ for sufficiently large positive integers n. (This means there exists an integer K so that for all $n \geq K$ the statement is true. What is K?)

Here is a similar problem: Show that, for sufficiently large integers n, $2^n > n^2$.

Answer: First I claim: If $n \geq 3$ then $2n^2 > (n+1)^2$. To prove this claim we first note that $n \geq 3$ implies

$$n^2 \geq 3n = 2n + n \geq 2n + 3 > 2n + 1$$

Adding n^2 to both sides we see that $2n^2 > n^2 + 2n + 1 = (n+1)^2$ as claimed.

Next, we note that the statement is true for $n = 5$ since $2^5 = 32 > 5^2 = 25$. Suppose by induction that $n \geq 5$ and $2^n > n^2$. Then

$$2^{n+1} = 2(2^n) > 2n^2 > (n+1)^2$$

where the first inequality holds by induction and the second inequality is the claim proved above. We conclude by induction that the statement holds for all integers $n \geq 5$. (But it is not true for $n = 2, 3, 4$.)

b) Let A be any set and let $C = \{0, 1, 2\}$. Find a bijection between the set C^A and the set of all pairs (X, Y) of subsets of A so that $X \subseteq Y$.

c) Let $f : A \to B$ be a function which is three-to-three, i.e., for any subset C of A with $|C| = 3$ the image also has three elements: $|f(C)| = 3$. When is f a 1-1 function? (Using the concept of vacuous truth.)

Key words: Induction, sufficiently large, bijection, 1-1, claim.