

1. MATH 30A, FALL 2009

Quiz 1

Rules: Closed book. 40 minutes. You may bring notes written on a single piece of paper. You need to decide what should be written on your paper.

Write all answers in the “blue book”. Write your real name on the cover of the exam booklet.

1.1. Explain why each of the following is NOT a group.

- (1) (\mathbb{Q}, \cdot) (rational numbers under multiplication)
- (2) The set of positive real numbers under the operation $a * b = e^{ab}$.
- (3) The subset H of $GL(2, \mathbb{R})$ given by the equation:

$$H := \{A \in GL(2, \mathbb{R}) \mid \det A \in \mathbb{Z}^+\}$$

with operation given by matrix multiplication.

1.2. Draw the Cayley digraph for the group \mathbb{Z}_6 with generating set $\{3, 4\}$. Make sure the vertices and edges are appropriately labeled. Use your diagram to explain something about the group.

1.3. Find all cyclic subgroups of the symmetric group on 3 letters. How many are there?

1.4. Give an example of an infinite group G with a subgroup H having 4 elements.

1.5. Prove that a group in which every element (except the identity) has order 2 is abelian.

- (1) First restate the problem using the definitions and with some choice of notation. What does this question ask for?
- (2) Make sure to justify each step in your proof or calculation.