1. MATH 30A, FALL 2009
QUIZ 1 ANSWERS TO PRACTICE 1.1

1.1. **Practice for quiz 1.** Rules: Closed book. 15 minutes. You may bring notes written on a single piece of paper. You need to decide what should be written on your paper.

Don't hand in a blank sheet of paper. Use an alias if you like.

1.1.1. List all the subgroups of $\mathbb{Z}_6$. How many are there?

The group $\mathbb{Z}_6$ has 4 subgroups: The trivial group $\{e\}$, the whole group $\mathbb{Z}_6$ and

$$\langle 2\rangle = \{0, 2, 4\}$$
$$\langle 3\rangle = \{0, 3\}$$

The key point to remember is: Every subgroup of a cyclic group is cyclic. But $\langle g\rangle$ might be equal to $\langle h\rangle$.

OK, now find all the subgroup of V.

1.1.2. Give an example of two groups which are not isomorphic and explain why they cannot be isomorphic. The groups $\mathbb{Z}_2$ and $\mathbb{Z}_3$ are not isomorphic because they have different numbers of elements. When it says "Give and example" you need to give an explicit example. Also, many of you gave me examples which are not groups!!

OK, now find two groups of the same order which are not isomorphic. In order to show that two groups are not isomorphic, you need to find a structural property of one which is not held by the other. Looking at all possible mappings $\phi : G_1 \to G_2$ just does not work.

1.1.3. Prove that $(\mathbb{Q}, +)$ is not a cyclic group.

The proof is by contradiction. Suppose that $\mathbb{Q}$ were cyclic. Then it would be equal to $\left\langle \frac{a}{b} \right\rangle$ for some rational number $\frac{a}{b}$. But, the elements of the cyclic group $\left\langle \frac{a}{b} \right\rangle$ are $\frac{na}{b}$ which have denominator $\le b$. So, $\frac{1}{2b}$ is not in this set. So, we get a contradiction. So, $\mathbb{Q}$ is not cyclic.

One point about notation: We write g^n when we don't know what the operation is. If we know the operation is addition, as in this case, we write ng for the nth additive power and $-g$ for the additive inverse. Please use the following definition of rational numbers:

$$\mathbb{Q} := \left\{ \frac{a}{b} \,|\, a \in \mathbb{Z}, b \in \mathbb{Z}^+ \right\}$$

where $\frac{a}{b} = \frac{c}{d}$ if $ad = bc$.

Now try this: Prove that $(\mathbb{Q}, +)$ is not generated by any finite set S.