

2. MATH 30A, FALL 2009

**Quiz 2**

Rules: Closed book. 45 minutes. You may use notes written on a single piece of paper. This can be one of the handouts as long as it is only one sheet of paper.

Choose 4 questions and answer them using *complete sentences*. (For example: “ $H$  has 4 left cosets and they are ...”)

2.1. Assume that rings have been defined.

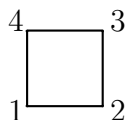
- (1) What is the definition of a field?
- (2) Given an example of a ring with unity which is not a field and explain which part of the definition fails.

2.2. What are the elements of the factor group  $S_5/A_5$ ? Explain in detail how they are multiplied.

2.3. Give the statement of LaGrange’s theorem and explain why it implies that the order of an element divides the order of a finite group. Give an example to illustrate the formula.

2.4. The dihedral group  $D_4$  acts on the set  $\{1, 2, 3, 4\}$  in the usual way. (These are the corners of the unit square and  $D_4$  is the group of rotations and reflections of the unit square.)

- (1) Find the stabilizer subgroup of vertex 2. You don’t have to prove it. Just tell me exactly which subgroup it is.
- (2) What is the orbit-stabilizer formula. What are the numbers in this particular case and what things do the numbers count in this particular case? (Alternative question: What is the orbit-coset correspondence in general and tell me exactly which elements of which sets correspond to which elements of which other set in this particular case?)



2.5. Prove that a cyclic group  $\mathbb{Z}_n$  is simple if and only if  $n$  is a prime number.