2. MATH 30A, FALL 2009

Some answers for Quiz 2

2.4. The dihedral group D_4 acts on the set $\{1, 2, 3, 4\}$ in the usual way. (These are the corners of the unit square and D_4 is the group of rotations and reflections of the unit square.)

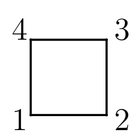
The elements of the group D_4 are the rotations e, t, t^2, t^3 and reflections $s = (12)(34), st = (24), st^2 = (14)(23), st^3 = (13)$.

(1) Find the stabilizer subgroup of vertex 2. You don't have to prove it. Just tell me exactly which subgroup it is. Only $st^3 = (13)$ and the identity fix 2 so, $G_2 = \langle st^3 \rangle = \{e, st^3\}$.

(2) What is the orbit-stabilizer formula. It says that the index of the stabilizer is the size of the orbit:

$$|Gx| = \frac{|G|}{|G_x|}$$

What are the numbers in this particular case and what things do the numbers count in this particular case? The numbers are $|G_2| = 2, |G| = 8, |G2| = 4$. The stabilizer of 2 has two elements (listed above). The dihedral group has 8 elements and the orbit of 2 is the entire set of vertices: $\{1, 2, 3, 4\}$. (Alternative question: What is the orbit-coset correspondence in general and tell me exactly which elements of which sets correspond to which elements of which other set in this particular case? This says that there is a 1-1 correspondence between the elements of the orbit of x and the left cosets of the stabilizer G_x. The correspondence is given by $y = gx \leftrightarrow gG_x$.)



2.5. Prove that a cyclic group $\mathbb{Z}_n$ is simple if and only if n is a prime number. We did this in class. If n is not prime, say $n = ab$ where $a, b \geq 2$ then $a\mathbb{Z}_n = \langle a \rangle$ is a proper subgroup of $\mathbb{Z}_n$. Since all subgroups of an abelian group are normal, this subgroup is a nontrivial proper normal subgroup and $\mathbb{Z}_n$ is not simple.

Any subgroup H of $\mathbb{Z}_n$ has order dividing n. If n is prime then $|H|$ must be either n or 1. So, the only subgroups of $\mathbb{Z}_n$ are the trivial group and the whole group. So $\mathbb{Z}_n$ is simple for n prime.

2.2. What are the elements of the factor group S_5/A_5? Explain in detail how they are multiplied.

The factor group S_5/A_5 has exactly two elements. They are

(1) $E := A_5 =$ the set of even permutations in S_5
(2) $O := (12)A_5 =$ the set of odd permutations in S_5

These two elements, which we are denoting E, O are multiplied as follows. $EE = E, EO = O, OE = O, OO = E$.