

2. MATH 30A, FALL 2009
Review for Quiz 2

Rules for the quiz: Closed book. 45 minutes. Do 4 out of 5. (I already wrote the quiz. It has one proof which we did in class.) *You may bring notes written on a single piece of paper (both sides, letter size). You need to decide what should be written on your paper. You can't ask me about definitions. I am going to ask you! Also statements of theorems.*

2.1. List of topics.

Sections 9 through 18. (Section 18 is the first section about rings and fields.)

cycles and permutations How many elements of order 5 are in the group S_5 ? Show that they are all conjugate. (What is the definition of *conjugate*? This is from section 15.)

Alternating group A_n What is the definition of A_n . Why is $A_n \trianglelefteq S_n$?

cosets List all left cosets of $\langle(12)\rangle$ in S_3 . Find all cosets of $3\mathbb{Z}$ in \mathbb{Z} . What is the index of \mathbb{Z}_n in $\mathbb{Z}_n \times \mathbb{Z}_m$?

Lagrange What is the exact statement of Lagrange's Theorem.

direct product of groups If $a \in G$ has order n and $b \in H$ has order m . What is the order of $(a, b) \in G \times H$.

homomorphisms If $g \in G$ then show that

$$\phi : \mathbb{Z} \rightarrow G$$

given by $\phi(n) = g^n$ is a homomorphism. Suppose that $o(g) = 6$. Then what is the kernel and image of ϕ ? What does the isomorphism theorem say about this?

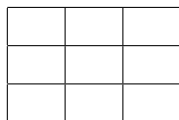
factor groups What are the elements of the factor group $GL(2, \mathbb{R})/Z$ where Z is the center of $GL(2, \mathbb{R})$ which is the group of all nonzero scalar multiples of the identity matrix

$$Z = \left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} \text{ s.t. } x \in \mathbb{R}^\times \right\}$$

What is the difference between \mathbb{Z}_n and $\mathbb{Z}/n\mathbb{Z}$?

simple groups What is the definition of a simple group? Prove that the center of a nonabelian simple group is trivial.

group actions Take a 3×3 square:



How many ways can you color this using 3 colors on the 9 squares is

- (1) you only color one side of the square?
- (2) if the square is made of glass so you color both sides the same color?

In each case, what is the group G and what is the set X ?

rings and fields

- (1) general rings. Why is $GL(2, \mathbb{R})$ not a ring?
- (2) commutative rings. What is the definition. Give an example of a noncommutative ring.
- (3) rings with unity. Give an example of a ring without unity.
- (4) units in a ring with unity. Find all the units in \mathbb{Z} . Find all the units of $\mathbb{Z} \times \mathbb{Z}$.
- (5) homomorphisms. Find a homomorphism $\mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ and determine its kernel.
- (6) fields. Show that \mathbb{C} is a field. Note: we always assume all the usual properties like $a + b = b + a$, $(ab)c = a(bc)$. So, what do think you actually need to show?

More later.