2. Math 30a, Fall 2009
2nd Review for Quiz 2

Rules: Closed book. 45 minutes. You may use notes written on a single piece of paper. This can be one of the handouts as long as it is only one sheet of paper.

Choose 4 questions and answer them using *complete sentences*. (For example: "H has 4 left cosets and they are ...")

2.1. List of topics.

Sections 9 through 18. (Section 18 is the first section about rings and fields.) (We did not cover sections 11,12 except for the definition of a product of groups and the theorem $\mathbb{Z}_{nm} \cong \mathbb{Z}_n \times \mathbb{Z}_m$ if n, m are relatively prime which are from section 11.)

cycles and permutations (see group actions question)

Alternating group A_n If σ, τ are elements of S_n and their product $\sigma\tau$ is not in A_n then show that either σ or τ ?

cosets Find a bijection between the set of left cosets of H in G and the set of right cosets. When are two elements of G contained in the same left coset of H? Give an example to illustrate this idea using two particular elements (your choice) of the group $GL(2, \mathbb{R})$ and as subgroup the center of $GL(2, \mathbb{R})$.

Lagrange and related concepts What is the definition of the *index* of a subgroup H of a group G? Give an example of group G and a subgroup H of index 4.

direct product of groups If $N \trianglelefteq G$ and $M \trianglelefteq H$ then show that $N \times M \trianglelefteq G \times H$. Determine the order of every element of the group $\mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_5$.

homomorphisms Give an example of a homomorphism $\phi : G \to H$ which is neither 1-1 nor onto. Find the kernel and image of your homomorphism. What does the isomorphism theorem tell you about your example. (Can you find one answer using abelian groups and one using nonabelian groups?)

factor groups Let S be the set of all integers (positive or negative) whose last digit is 5. Then S is one element of a certain factor group. What is that factor group? How many elements does it have? Is there more than one correct answer to this question?

simple groups What is the definition of a normal subgroup? Is it possible for a product $G \times H$ to be a simple group?

group actions Let $G = \langle \sigma \rangle$ be the subgroup of S_5 generated by the element $\sigma = (123)(45)$ in cycle notation. Consider the action of G on $X = \{1, 2, 3, 4, 5, 6\}$

(1) How many orbits does this action have.
(2) What is the stabilizer of 5?
(3) List the permutations in the coset of G corresponding to 4 under the coset-orbit correspondence.

rings and fields

(1) general rings. Why is $\{1, 2, 3, 4\}$ not a subring of $\mathbb{Z}_5$?
(2) If
(3) rings with unity. Prove that $(1, 1)$ is unity in $R \times S$.
(4) units in a ring with unity. Find all elements of $\mathbb{R} \times \mathbb{R}$ which are not units.
(5) homomorphisms. If $\phi : R \to S$ is a homomorphism of rings and R is commutative, it is necessarily true that S is commutative?
(6) fields. Which of the following rings are fields? which are commutative rings which are not fields? Is there anything in the list which is not a ring?
   (a) $\{a + b\sqrt{3} \,|\, a, b \in \mathbb{Q}\}$ with the usual operations.
   (b) The set of all diagonal 2×2 matrices with real coefficients:
   $$\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}, \quad x, y \in \mathbb{R}$$
   under matrix multiplication and addition.
   (c) $\{1, 2, 3, 4\}$ under addition and multiplication modulo 5.