ANOVA Finding totals and averages

treatment		Drug A	Drug B	Drug C
column #	j	1	2	3
sample size	n_j	7	8	10
mean	$\overline{Y}_{\bullet j}$	80	88	90
variance	S_j^2	5.2	4.8	5.4

The total for each treatment is given by

$$T_{\bullet j} = n_j \overline{Y}_{\bullet j}$$

With grand total $T_{\bullet\bullet} = \sum T_{\bullet j}$:

$$T_{\bullet\bullet} = \sum_j n_j \overline{Y}_{\bullet j} = 7 * 80 + 8 * 88 + 10 * 90$$

$$= 560 + 704 + 900 = 2164$$

$$Y_{\bullet\bullet} = \frac{1}{N} T_{\bullet\bullet} = \frac{2164}{25} = 86.56$$

Finding SSTr, MSTr

The treatment sum of squares is

$$SS_{Tr} = \sum_i n_j (\overline{Y}_{\bullet j} - \overline{Y}_{\bullet\bullet})^2$$

This number tends to be big if the treatments are different. It is small when the treatments have no effect.

$$\begin{aligned} SS_{Tr} &= 7 * (80 - 86.56)^2 \\ &+ 8 * (88 - 86.56)^2 \\ &+ 10 * (90 - 86.56)^2 \\ &= 436.16 \end{aligned}$$

The number of degrees of freedom is the number of treatments minus 1:

$$df = k - 1 = 2$$

The formula for MS_{Tr} is

$$MS_{Tr} := \frac{SS_{Tr}}{k-1} = \frac{436.16}{2} = 218.08$$

Finding SSE, MSE

The error sum of squares is

$$SS_E = \sum_i \sum_j (Y_{ij} - \overline{Y}_{\bullet j})^2$$

$$= \sum_j (n_j - 1) S_j^2$$

This number measures random errors and variability of data. It tell us nothing about the treatments (drugs in this case).

$$SS_E = 6(5.2) + 7(4.8) + 9(5.4) = 113.4$$

The degrees of freedom is

$$df = \sum_j (n_j - 1) = N - k = 25 - 3 = 22$$

So the mean square error is:

$$MS_E = \frac{SS_E}{N-k} = \frac{113.4}{22} = 5.15$$

ANOVA, F-test

The test statistic is

$$F_{(n_1-1,n_2-1)} = \frac{MS_{Tr}}{MS_E}$$

$$F_{(2,22)} = \frac{218.08}{5.15} = 42.3$$

In ANOVA the F-test is always right tailed. When F is large we conclude that there is a significant difference between the drugs. This is because the numerator measures the difference between treatments.

The critical value is

$$F_{2,22,.95} = 3.44$$

Since the data value of F is much larger than critical we conclude that the drugs are different. But we don't know if they make people better or worse.