MATH 36B: HOMEWORK

1. Homework 01

1.1. **page 356 # 5.2.12.** If the random variable Y denotes an individual's income, Pareto's law claims that $\mathbb{P}(Y \geq y) = (k/y)^\theta$ where k is the entire population's minimum income. It follows that $F_Y(y) = 1 - (k/y)^\theta$, and, by differentiation,

$$f_Y(y;\theta) = \theta k^\theta \left(\frac{1}{y}\right)^{\theta+1}, \quad y \geq k; \theta \geq 1$$

Assume k is unknown. Find the maximum likelihood estimator for θ if income information has been collected on a random sample of 25 individuals.

[This means that your data consists of the income $Y_1, Y_2, \cdots, Y_{25}$ from 25 people chosen at random. The estimator is the formula for $\hat{\theta}$ using these 25 numbers.]

1.2. **page 363 # 5.2.20.** Find a formula for the method of moments estimate for the parameter θ in the Pareto pdf

$$f_Y(y;\theta) = \theta k^\theta \left(\frac{1}{y}\right)^{\theta+1}, \quad y \geq k; \theta \geq 1$$

Assume k is known and the data consist of a random sample of size n. Compare your answer to the MLE found in 5.2.12.

1.3. **page 387 # 5.4.2.** Suppose a random sample of size $n = 6$ is drawn from the uniform pdf $f_Y(y;\theta) = 1/\theta, 0 \leq y \leq \theta$ for the purpose of using $\hat{\theta} = Y_{max}$ to estimate θ.

(a) Calculate the probability that $\hat{\theta}$ falls within 0.2 of θ given that the parameter's true value is $\theta = 3.0$. [Hint: $\mathbb{P}(Y_{max} \leq y) = \mathbb{P}(Y_i \leq y)^n$.]

(b) Calculate the probability of the event asked for Part (a) assuming the sample size is 3 instead of 6.

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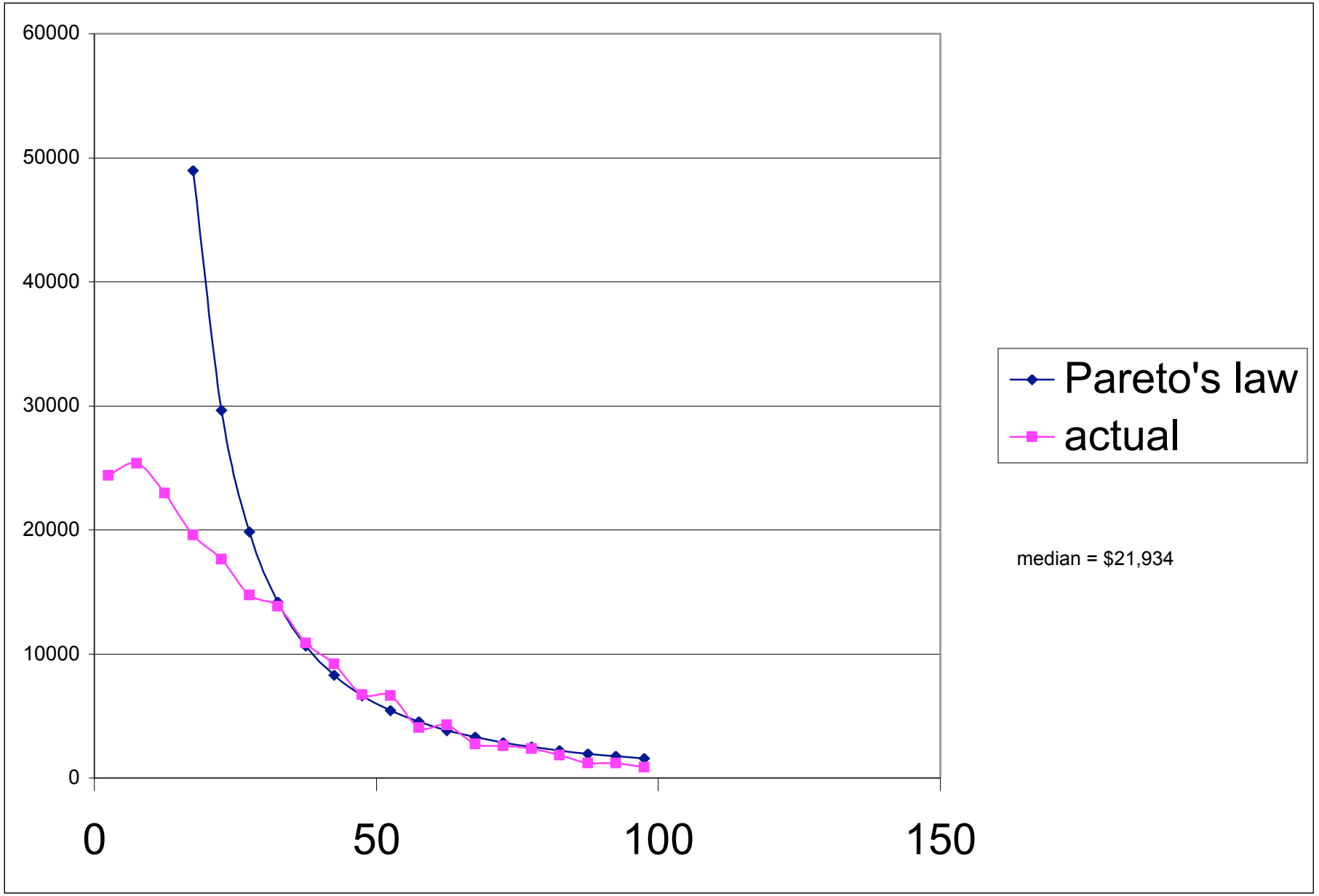
1.4. **page 387 # 5.4.10.** A sample of size 1 is drawn from the uniform pdf defined over the interval $[0, \theta]$. Find an unbiased estimator for θ^2. Hint: Is $\hat{\theta} = Y^2$ unbiased? [Another hint: in general $\hat{\theta}^2 \neq \widehat{\theta^2}$, in other words, the square of an unbiased estimator for θ is usually a biased estimator for θ^2. For example, the formula for the sample standard deviation S is a biased estimator of σ, but its square S^2 is an unbiased estimator for σ^2. This exercise is supposed to get you to see this for yourself.]

The next page compares the income distribution for families in the US with the Pareto distribution. At the lower end, there are fewer families with low income (less than 30k) than predicted by Pareto. This is presumably a result of Welfare.


60000
50000
40000
30000
20000
10000
0
0
50
100
150
Pareto's law
actual
median = $21,934