

REVIEW FOR QUIZ 2

1. Basic properties of \mathbb{E} and Var .

- (1) Given that X, Y are independent random variables calculate

$$\mathbb{E}((X + Y)^2)$$

in terms of $\mu_X, \mu_Y, \sigma_X, \sigma_Y$. These Greek letters are defined to be the mean $\mu_X = \mathbb{E}(X)$ and standard deviation $\sigma_X^2 = \text{Var}(X)$ of the variables.

- (2) If $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators for θ_1, θ_2 then show that $2\hat{\theta}_1 + 3\hat{\theta}_2$ is an unbiased estimator for $2\theta_1 + 3\theta_2$.

2. MLE and MOM

- (1) Given that X has continuous distribution $f(x) = e^{\theta-x}$ for $x \geq \theta$ and $f(x) = 0$ for $x < \theta$ find the MOM estimator for θ . Given a sample of two: $X_1 = 5, X_2 = 35$ find the MLE for θ .
- (2) Given that X is a discrete random variable taking only the three values 0, 1, 2 with probability

$$\mathbb{P}(X = 0) = \theta, \quad \mathbb{P}(X = 1) = 2\theta, \quad \mathbb{P}(X = 2) = 1 - 3\theta$$

If you take a sample of $n = 4$ and get the numbers 0, 2, 0, 1 find the MLE for θ .

3. Bias and efficiency

Are the estimators that you got in the previous problem unbiased? If so, find the efficiency.

4. You take a sample of 25 from an exponential distribution and get $\bar{T} = 2.3$. Find the 95% confidence interval for λ .

5. Suppose that X has density function $f(x) = 2\lambda e^{-\lambda x^2}$ if $x \geq 0$ and $f(x) = 0$ for $x < 0$. Then find the Cramér-Rao lower bound for the variance of any unbiased estimator for λ .