0.3. **some answers.** Here are some quick answers.

(1) definitions: What are the definitions of the F, t, χ^2 distributions?

(a) $F_{\nu,\mu} = \frac{\chi^2_\nu/\nu}{\chi^2_\mu/\mu}$

(b) $t_\nu = \frac{Z}{\sqrt{\chi^2_\nu/\nu}}$

(c) $\chi^2_\nu = Z_1^2 + \cdots + Z_\nu^2$

(2) F-distribution: You have two populations of size n, m. The sample deviations are S_X, S_Y. You want to test is S_Y is greater than S_X with 90% confidence. How do you proceed? The null hypothesis is $H_0 : \sigma_X = \sigma_Y$ with alternate hypothesis $H_a : \sigma_X < \sigma_Y$. Use the F-distribution to test this:

$$F_{n-1,m-1} = \frac{S_X^2}{S_Y^2}$$

You look up the critical value $F_{n-1,m-1,.10}$ (make sure that it is less than one). If S_X^2/S_Y^2 is less than this critical value then you reject the null hypothesis. Find the pooled variance. This is given by

$$S_p^2 = \frac{(n-1)S_X^2 + (m-1)S_Y^2}{n+m-2}$$

Suppose that you decide that $\sigma_X = \sigma_Y$ then what is the formula for the the 90 confidence interval for the common standard deviation? Use the χ^2 distribution:

$$\chi^2_{n+m-2} = \frac{(n+m-2)S_p^2}{\sigma^2}$$

You look up the two critical values $a = \chi^2_{n+m-2,.05}, b = \chi^2_{n+m-2,.95}$. (So, $a < b$) Then the 90% confidence interval will be

$$\sqrt{\frac{(n+m-2)S_p^2}{b}} \leq \sigma \leq \sqrt{\frac{(n+m-2)S_p^2}{a}}$$

(3) t-distribution Suppose that you have two populations of size $5, 7$. You got $\overline{X}, \overline{Y}$. You calculated the pooled variance $S_p^2 = 25$. What is the critical value of $\overline{X}, \overline{Y}$ to test if $\mu_X = \mu_Y$ against the three possible alternate hypotheses? You use the t-distribution:

$$t_{10} = \frac{\overline{X} - \overline{Y}}{5\sqrt{\frac{1}{5} + \frac{1}{7}}} = \frac{\overline{X} - \overline{Y}}{2.9277}$$

You look up the critical values $a = t_{10,1-\alpha}$ and $b = t_{10,1-\alpha/2}$

(a) If $|\overline{X} - \overline{Y}| > 2.9277b$ you reject H_0 in favor of $H_1 : \mu_X \neq \mu_Y$.

(b) If $\overline{X} - \overline{Y} > 2.9277a$ you reject H_0 in favor of $H_1 : \mu_X > \mu_Y$.

(c) If $\overline{X} - \overline{Y} < -2.9277a$ you reject H_0 in favor of $H_1 : \mu_X < \mu_Y$.

(4) χ^2-distribution

(a) to determine σ_X If $S_X^2 = 100$ and $n = 6$ find the 90% confidence interval for σ_X Look up the critical values $\chi^2_{5,.05}$ and $\chi^2_{5,.95}$. Then solve the equation $\chi^2_5 = 5S_X^2/\sigma_X^2$ for σ_X^2 and insert the two critical values of χ^2_5.

(b) for goodness of fit Test if the the numbers 3,8,10,11 are uniformly distributed?
The expected values are all 8. So,

$$\chi_3^2 = \sum_{i=1}^{4} \frac{(O_i - 8)^2}{8} = \frac{25 + 4 + 9}{8} = 4.75$$

Since this is less than the critical value $\chi^2_{3,.95} = 7.815$ we cannot reject the hypothesis that these numbers are uniformly distributed.

(5) z-distribution (can't think of anything)

(6) Hypothesis testing

(a) Type I and Type II errors: What do these mean? Type I means you reject that null hypothesis when it is true. The probability of this is α. Type II means you accept the null hypothesis when the alternate hypothesis is true. The probability of this is β.

(b) significance α: Why does $1-\beta$ converge to α? When the null hypothesis is very close to being true then $1-\beta$ which is the probability that the null hypothesis will be correctly rejected is close to α, the probability of the same event when the null hypothesis is true.

(c) power of test $1-\beta$ What is the definition of power? This is the probability that you correctly reject the null hypothesis when a specific alternate hypothesis is true. You are testing the hypothesis that $\sigma_X = 10$ vs $\sigma_X > 10$ at 90% confidence. Your sample size is 11. What is the power of the test if $\sigma_X = 12$? The answer is 35% but you can't do this without a computer or a detailed χ^2 chart. However, you can approximate. The critical value of S_X^2 is 159.87 Under the alternate hypothesis, we have

$$\chi_{10}^2 = \frac{10 S_X^2}{144} = 11.1$$

So, the power is the probability that $\chi_{10}^2 > 11.1$ which is .35 (The approximation is that, since χ_{10}^2 has an average value of 10, it will be more than 11 a little less than 50% of the time.)