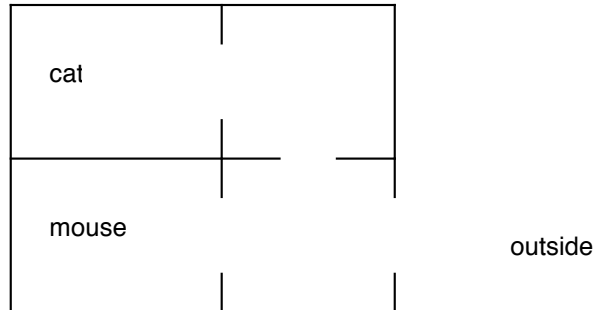


MATH 56A: STOCHASTIC PROCESSES
TAKE HOME FINAL EXAM

This is a take home exam. Please work in groups of 2 to 4. Write your answers clearly with complete details. Also from the syllabus: each student should write a confidential report: If I gave you \$100 for your project, how would you split up this fee with your teammates? If you don't write a report I will assume that you want to split it evenly.

This take-home exam is due on May 14 except that senior grades are due May 7! So, if you are a senior, you need to give me something by May 7.

Problem 1 There is a house with 4 rooms:



The mouse starts in the lower left room and wanders around, choosing a door at random (with equal probability). What is the probability that he will escape the house?

Problem 2

There are four cities: A, B, C, D . People move from city B to city A at the rate of 5%. They move from C to A at the rate of 10%. They go from D to B at the rate of 25% and from D to C at the rate of 5%. They also go from A to D at the rate of 1%. Find the infinitesimal generator. Find the probability transition matrix P_t What is the probability that a person in city D will end up in city A at the end of 2 years?

Problem 3 You go to a casino where there is something wrong with the roulette wheel. It comes up red 51% of the time! You have \$1000 and you bet \$50 each time (on red of course). You gain \$50 if you win and you lose \$50 if not. Is this Markov chain transient? recurrent? positive recurrent? null recurrent? What is the probability that you play forever?

Problem 4 Find the time reversed process for problems 2 and 3 if this makes sense.

Problem 5 $dZ_t = X_t dW_t + Y_t dt$, $f(t, x) = x^t$.

Find $df(t, Z_t)$.

Problem 6 Show that Z_t (as above) is a martingale if and only if $Y_t = 0$ for all t . Why is it not enough to have $\mathbb{E}(Y_t) = 0$?

Problem 7 Find all values of the constants a, b so that $e^{aW_t + bt}$ is a martingale.

Problem 8 Do problem 9.4 from the book. [Note the misprint: $\phi(z) = (2\pi)^{-1/2}e^{-z^2/2}$ is the density for $N = N(0, 1)$.]