

## MATH 56A: STOCHASTIC PROCESSES QUIZ

Make a team of 2-4 people. Rules for auditors: There must be at least two people registered in the class in each team. There can be at most one person not registered in the class on each team. Furthermore, this unofficial person is only allowed to make suggestions and participate in a discussion about the problems. He/she cannot work out the answer to the question!!

Each team hands in only one quiz. Write your names and email addresses on the quiz.

Finally, give a *name* to your team and write it on the quiz. (So that I can refer to the team on the answer sheet.)

Also, *explain* where you get your answers.

You may email attachments to: igusa@brandeis.edu but you need to do this during the quiz.

### Quiz 1

1. You are given the following transition matrix.

$$P = \begin{pmatrix} .8 & 0 & 0 & 0 & .2 \\ 0 & .6 & 0 & .4 & 0 \\ .1 & .2 & 0 & .3 & .4 \\ 0 & .5 & 0 & .5 & 0 \\ .3 & 0 & 0 & 0 & .7 \end{pmatrix}$$

- a) Find all the communication classes. Which ones are recurrent? Number them  $R_1, R_2, \dots$
- b) Draw the diagram with dots and arrows representing this Markov chain.
- c) Find the invariant distribution for each recurrent class. Call them  $\pi_1, \pi_2, \dots$
- d) Find all invariant distributions.
- e) If your initial distribution is  $(.2, .2, .2, .2, .2)$ , what is the probability that you will end up in the recurrent class  $R_1$  ?

2. There is an epidemic and you are one of the susceptible people. From the following information construct a Markov chain.

- (1) Every day, susceptible people have a 1% chance of becoming infected.
- (2) On the first day of infection, you have a 10% chance of fully recovering the next day.
- (3) On the second day of infection, you have a 20% chance of fully recovering the next day.
- (4) On the third day of infection there is good news and bad news. The good news is that you will recover the next day. The bad news is that you have a 90% probability of being scarred forever!
- (5) More bad news: If you fully recover fully (without a scar) you are again susceptible!

Construct a Markov chain. Draw a diagram. Answer the following question:

What are the long term chances of you remaining unscarred?