MATH 56A: STOCHASTIC PROCESSES
QUIZ 2

Instructions:
1) Form groups. They cannot be identical to the groups from Quiz 1.
2) Don't shout out the answers!

Problem 1 ["Double or nothing"] Suppose that $S = \{0, 1, 2, 3, 4, \cdots\}$ with transition probabilities:

$$p(n, 2n) = \frac{2}{3}, \quad p(n, 0) = \frac{1}{3}, \quad p(0, 1) = 1$$

and all other $p(n, m) = 0$.

a) If $X_0 \neq 0$ what is the probability that $X_1, X_2, \cdots, X_n$ are all nonzero? Conclude that, with probability one, you will eventually reach 0.

b) Find the communication classes. (Recall that x, y are in the same class if you can get from x to y and from y to x.)

c) Which communications classes are transient, which are null recurrent and which are positive recurrent?

d) Find the invariant distribution for each positive recurrent communication class.

e) If $X_0 = 0$ find the expected return time to 0.

Problem 2 [Hint: look on page 78 of your book.] Consider the continuous birth-death chain with birth and death rates:

$$\lambda_n = \frac{1}{n+1}, \qquad \mu_n = 1$$

a) Show that this chain is positive recurrent.

b) Find the invariant distribution $\pi(n)$.

c) Find $A = (\alpha(n, m))$ (What is the formula for $\alpha(n, n)$?) and verify that your invariant distribution π is a left null eigenvector of the infinitesimal generator A. (i.e., check to see if your answer is correct!)