

MATH 56A: STOCHASTIC PROCESSES
QUIZ 3

This is a take home quiz. Please work in groups of 2 to 4. Write your answers clearly with complete details.

This take-home quiz is due on Thursday, April 10.

Problem 1 This is an example where subtracting the expected value does not give a martingale. (However, your answer might be different!)

Suppose that $X_i = 1, 2, 3$ with equal probability. Let

$$S_n = X_1 + X_2 + \cdots + X_n$$

- a) Find the expected value of S_n^2 and let M_n be equal to S_n^2 minus your answer.
- b) Is M_n a martingale or not? Prove it (with a calculation).
- c) Let $W_n = S_n^2 - \mathbb{E}(S_{n+1}^2 | \mathcal{F}_n)$. Is this a martingale? Prove it!

Problem 2 Flip a fair coin n times and let S_n be the number of heads.

- a) Find all possible values of the constant A so that $M_n = A^{S_n}$ is a martingale.
- b) Find all possible values of the constant B so that $W_n = S_n - B^{S_n}$ is a martingale.