This is a take home quiz. Please work in groups of 2 to 4. Write your answers clearly with complete details.
This take-home quiz is due on Thursday, April 10.

Problem 1 This is an example where subtracting the expected value does not give a martingale. (However, your answer might be different!)
Suppose that $X_i = 1, 2, 3$ with equal probability. Let

\[ S_n = X_1 + X_2 + \cdots + X_n \]

a) Find the expected value of $S_n^2$ and let $M_n$ be equal to $S_n^2$ minus your answer.
b) Is $M_n$ a martingale or not? Prove it (with a calculation).
c) Let $W_n = S_n^2 - \mathbb{E}(S_{n+1}^2 | \mathcal{F}_n)$.
Is this a martingale? Prove it!

Problem 2 Flip a fair coin $n$ times and let $S_n$ be the number of heads.
a) Find all possible values of the constant $A$ so that $M_n = A^{S_n}$ is a martingale.
b) Find all possible values of the constant $B$ so that $W_n = S_n - B^{S_n}$ is a martingale.