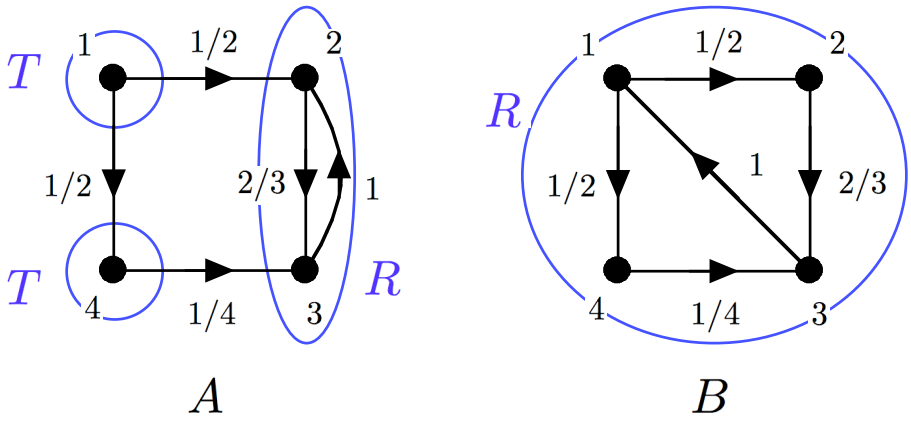
MATH 56A: STOCHASTIC PROCESSES
WORKSHEET

1. ANSWERS TO WORKSHEET 1



For each of these Markov chains:

(0) Draw in the implied loops. In both cases there is an implied loop at 2 with probability 1/3 and another one at 4 with probability 3/4.
(1) Circle the communication classes. in blue
(2) Label them R for recurrent and T for transient.
(3) Are the recurrent classes periodic? No, because of the loop at 2.
(4) Write down the transition matrix in each case.

$$P_A = \begin{pmatrix} 0 & 1/2 & 0 & 1/2 \\ 0 & 1/3 & 2/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/4 & 3/4 \end{pmatrix}, \qquad P_B = \begin{pmatrix} 0 & 1/2 & 0 & 1/2 \\ 0 & 1/3 & 2/3 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1/4 & 3/4 \end{pmatrix}$$

(5) Find $p_2(1,3)$ and $p_3(1,3)$. (The answer is the same for both A and B. Why is that?) In both cases,

$$p_2(1,3) = p(1,2)p(2,3) + p(1,4)p(4,3) = \frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{3} + \frac{1}{8} = \frac{11}{24}.$$

Also, in both cases, the only way to get from 1 to 3 in exactly 3 steps is if you stop one turn at the intermediate point (2 or 4). So

$$p_3(1,3) = p(1,2)p(2,2)p(2,3) + p(1,4)p(4,4)p(4,3) = \frac{1}{2}\cdot\frac{1}{3}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4} = \frac{1}{9} + \frac{3}{32} = \frac{59}{288}$$

The answer is the same in both cases because the only difference between them is the transition probability from the point 3. But, if you use this arrow (the one with probability one) then it will take you at least 4 steps to get from 1 to 3.