

MATH 56A: STOCHASTIC PROCESSES WORKSHEET

2. WORKSHEET 2

The purpose of this worksheet is to study Markov chains using only linear algebra. So, don't draw the graph until the end!

Please discuss the questions with your neighbors. Compare answers to see if they agree!

$$P = \begin{pmatrix} .8 & 0 & .2 & 0 & 0 \\ 0 & .5 & 0 & 0 & .5 \\ .3 & 0 & .7 & 0 & 0 \\ .2 & 0 & 0 & .6 & .2 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

- (1) Calculate the rank of $P - I_5$ by *column reduction*.
- (2) How many recurrent classes does this Markov chain have?
- (3) Find the left null vectors of $P - I_5$. Normalize to get the *basic invariant distributions*.
- (4) What are the *supports* of these vectors? These are the recurrent classes.
- (5) Find all invariant distributions π .
- (6) If the initial distribution is

$$\alpha = \left(0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0 \right),$$

what is the invariant distribution

$$\alpha P^\infty := \lim_{n \rightarrow \infty} \alpha P^n \quad ?$$

- (7) Renumber the states and put the matrix P into *canonical form*.