

MATH 56A: STOCHASTIC PROCESSES HOMEWORK

From the syllabus: There will be weekly homework. The first HW might have the problem: Using the SIR model, prove that the number of infected reaches its highest point when the size of the susceptible population reaches the threshold. Students are encouraged to work on their homework in groups and to access all forms of aid including expert advice, internet and other resources. The work you hand in should, however, be in your own words and in your own handwriting. [Or, from your computer.] And you should understand what you have written. Every student hands in his own homework. [and receives a separate grade]

Homework will be numbered according to the chapter in the book that they refer to. So, I plan to assign 10 homeworks numbered 0-9.

There will be a penalty for late homework. Homework which is more than one week late will not be accepted.

0. HOMEWORK 0 LINEAR DIFFEQ'S AND DIFFERENCE EQUATIONS

Four problems due next Monday, Jan 28:

0.1. In the Kermack-McKendrick model, prove that the number of infected reaches its highest point when the number of susceptibles is equal to the threshold (or at $t = 0$ at the beginning of the recorded/modeled time period).

0.2. Find all functions $x(t), y(t)$ so that

$$x'(t) = 5x - y$$

$$y'(t) = 3x + y$$

Find the particular solution with initial position $(x_0, y_0) = (1, 3)$.

0.3. Find all functions f from integers to complex numbers so that

$$f(n+1) = 4f(n) - 5f(n-1).$$

Now find the solution when $f(0) = f(1) = 2$ and explain why it is real.

0.4. (From the book)

Find the function $f(n), n = 0, 1, 2, 3, \dots$ so that $f(0) = 0$

$$f(n) = \frac{1}{3} [f(n-1) + f(n+1) + f(n+2)], \quad n \geq 1$$

$$\lim_{n \rightarrow \infty} f(n) = 1.$$