

MATH 56A: STOCHASTIC PROCESSES HOMEWORK

From the syllabus: There will be weekly homework. Students are encouraged to work on their homework in groups and to access all forms of aid including expert advice, internet and other resources. The work you hand in should, however, be in your own words and in your own handwriting. Every student hands in his own homework. There will be a penalty for late homework.

HOMEWORK 1A FINITE MARKOV CHAINS

Three problems (corrected) due next Monday, Feb 4:

0.1. Do 1.5 in the book.

0.2.

A man is playing two slot machines (call them A and B). Machine A gives a payoff with a probability of $1/6$ and machine B gives a payoff with probability $1/16$. The man starts by picking a machine at random. Then he plays the machine until he has lost twice (not in a row, just in total). Then he switches machines and continues. For example, suppose that his winning (1) and losing (0) sequence might be:

$$01_10_20_31_40_51_60_71_80_91_{10}0_{11}0_{12}$$

Then he will switch machines after $n = 2$ since he lost twice. (He switches in the time between $n = 2$ and $n = 3$). He switches back after $n = 6$ and then again after $n = 10$.

- (a) Make this into a Markov chain with 4 states: A_0, A_1, B_0, B_1 where the subscript keeps track of the number of losses. [This is an example of recording information to convert a stochastic process to first order.]
- (b) What is the probability that the man will be playing machine A at $n = 4$ if he starts at machine A ? What about if he starts at a machine picked at random?
- (c) Find the invariant distribution.
- (d) In the long run, how much of the time will the man be playing the better machine?

0.3. Suppose that

$$P = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ .4 & .4 & 0 & .2 \\ .7 & 0 & 0 & .3 \end{pmatrix}$$

- (a) Find the unique invariant distribution and explain why it is unique.
- (b) Draw the diagram and find the communication classes.
- (c) What is the probability that X_{100} is in the transient class given that you start in the transient class? What about if you start at a random location?