

## MATH 56A: STOCHASTIC PROCESSES HOMEWORK

From the syllabus: There will be weekly homework. Students are encouraged to work on their homework in groups and to access all forms of aid including expert advice, internet and other resources. The work you hand in should, however, be in your own words and in your own handwriting. Every student hands in his own homework. There will be a penalty for late homework.

### HOMEWORK 3 CONTINUOUS MARKOV CHAINS

Three problems due 6pm Monday, March 10. Answers will be posted Tuesday evening.

Quiz 2 on Friday, March 14.

**3.4, 3.9** plus the following.

**(M/M/1 queue).**

Suppose that there is a queue with one server. People get into the line at a rate of  $\lambda$  and they get served at the rate of  $\mu$ .

- (1) This is a continuous Markov chain  $X_t$  with states  $0, 1, 2, 3, \dots$ . What is the infinitesimal generator  $A = (a(x, y))$ ?
- (2) Convert this to a countable Markov chain  $Z_n$ . What is the (infinite) probability transition matrix  $P = (p(x, y))$ ?
- (3) Using your answers to Homework Problem #2.1 determine
  - (a) Under what conditions is this queue transient, positive recurrent, null recurrent?
  - (b) When it is positive recurrent, what is the expected return time to 0? (For  $X_t$  not  $Z_n$ ).
- (4) Determine when the chain  $X_t$  is explosive.