

MATH 56A: STOCHASTIC PROCESSES HOMEWORK

From the syllabus: There will be weekly homework. Students are encouraged to work on their homework in groups and to access all forms of aid including expert advice, internet and other resources. The work you hand in should, however, be in your own words and in your own handwriting. Every student hands in his own homework. There will be a penalty for late homework. The penalty is 10 to 20 % per day depending on the difficulty of the homework.

HOMEWORK 4 OPTIMAL STOPPING TIME

Two problems due Wednesday, March 19. Answers will be posted the following week.

Quiz 2 postponed until Thursday, March 20.

4.6 with the following additional questions.

(c) Suppose there is a discount α but no cost. What is the *largest* value of α so that you should stop no matter what you get?

(d) Suppose that $g(x) = 5$ and $\alpha = .8$. Then what is the optimal strategy? Calculate the value function $v(x)$.

Hint: The iteration algorithm should start with:

$$u_1(x) = \begin{cases} 0 & \text{if recurrent} \\ f(x) & \text{if } f(x) \geq \max(\alpha f(y) - g(x)) \\ \max(\alpha f(y) - g(x)) & \text{otherwise} \end{cases}$$

Second problem: This is random walk with absorbing wall on the left and reflecting wall on the right.

$$\begin{array}{rcccccccc} x = & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ f(x) = & 0 & 1 & 2 & 5 & 6 & 21 & 19 \\ g(x) = & 22 & 2 & 2 & 2 & 1 & 1 & 1 \end{array}$$

- a) Draw a graph of $f(x)$ and connect the dots.
- b) Find the optimal strategy and value function $v(x)$ if there is no cost. Describe geometrically what is the algorithm when the right wall is reflecting. [Hint: consider the mirror image of the function on the reflecting wall.]
- c) Find the optimal strategy and value function if the cost $g(x)$ is given as above.
- d) Find the optimal strategy and value function if there is a discount $\alpha = .9$ and no cost.
- e) Find the optimal strategy and value function if there is a discount $\alpha = .9$ and cost $g(x)$ as given above.