

**MATH 56A: STOCHASTIC PROCESSES
HOMEWORK**

HOMEWORK 6
RENEWAL

These problems are due Thursday, April 3. Answers will be posted the following week.

First problem: Suppose that we have a renewal process with a uniform distribution

$$f(t) = \begin{cases} 1/10 & \text{if } 0 < t \leq 10 \\ 0 & \text{otherwise} \end{cases}$$

(i.e., we have a light bulb which lasts at most 10 days and can burn out at any time with equal probability during those 10 days)

You walk into the warehouse after one year. The light bulbs are distributed according to the equilibrium distribution.

- (1) What is the average age of a light bulb (in this equilibrium distribution)?
- (2) If you pick a light bulb and it has been burning for 4 days, what is the probability that it will burn for at least 4 more days.

Second problem: a) Calculate the probability distribution of $Z = X + Y$ if X, Y are exponential variables with rate 2,3 resp.

b) Calculate the probability distribution of $W = X - Y$. [Hint: $-Y$ is a random variable with density function $f(y) = 3e^{3y}$ for $y < 0$ and $f(y) = 0$ if $y \geq 0$.

Third problem: We have a water tower which gives water to a desert town. It has been empty for a few day. :(

Suppose that it showers from time to time. This event is a Poisson process with rate $\lambda = 1/7$ (once a week on average). When it showers it always dumps 5,000 gallons of water into the tank. The residents of this town use 1,000 gallons of water per day continuously (at a constant rate throughout the day when there is water).

Convert this into an $M/G/1$ queue and determine:

- (1) What is the meaning of X_n, Y_n, U_n, τ in this case?
- (2) Compute μ .
- (3) If there is a shower tomorrow, how long can the town expect to have water?