Here is a list of recent papers and how they are related.

(2a) (2d) (3c) (3e) (1) +3 ◆ K S (2b) +3 (2c) +3 K S (3d) +3 (5d) [3a] (4a) (5b) (5c) (3b) (5a) (4b) (4c)

(1) (with G. Todorov, K. Orr, J. Weyman) “Modulated semi-invariants”[?] develops the basic cluster theory of hereditary algebras from the point of view of semi-invariant pictures.

(2) Papers about the picture group.
(a) (with G. Todorov) “Pictures groups and maximal green sequences”[?]. This paper proves that, in finite type, maximal green sequences are in bijection with positive expressions for the Coxeter element of the picture group.
(b) (with G. Todorov) “Signed exceptional sequences ...”[?] defines the “cluster morphism category” and proves that its classifying space is a $K(G,1)$ for quivers of finite type where $G$ is the picture group of the quiver. A purely combinatorial version of the cluster morphism category, for type $A_n$, is given in [?].
(c) (with G. Todorov and J. Weyman) “Picture groups of finite type ...”[?] computes the cohomology of the picture group of type $A_n$ with any orientation.
(d) (reporting on work of Eric Hanson) “Are finite type picture groups virtually special?”[?] generalizes pictures and picture groups to all finite dimensional algebras using recent results of [?], [?], [?], [?].

(3) Pictures for tame quivers, also called “propictures”, e.g., Figure ??.
(a) (with T. Brüstle, S. Hermes, G. Todorov) “Semi-invariants pictures ...”[?] uses semi-invariant pictures to study MGSs and show that there are only finitely many MGSs for cluster-tilted algebras of tame type.
(b) (with S. Hermes) “The no gap conjecture for tame hereditary algebras”[?] proves the “no-gap conjecture” for these algebras using [?]. This leads to an easy proof of the quantum dilogarithm identity for MGSs for tame quivers. (Theorem ??.)
(c) (with G. Todorov and J. Weyman) “Periodic trees and semi-invariants”[?] shows that clusters of type $\tilde{A}_n$ can be represented by periodic trees.
(d) (with G. Todorov, M. Kim, J. Weyman) “Periodic trees and propictures”[?] defines “propictures” and the “propicture groups” which are inverse limits of picture groups. Figure ?? above is an example.
(e) (with M. Kim) “Cluster propictures of type $\tilde{A}_n$”[?] extends definitions and theorems of [?] to cluster-tilted algebras of type $\tilde{A}_n$.

(4) Papers on the “linearity” question for maximal green sequences. (See Section ??.)
(a) “Linearity of stability conditions”[?] gives many equivalent definitions of maximal green sequences (mostly well-known) for hereditary algebras using the corresponding Harder-Narasimhan filtration. (See Theorem ?? above.)
(b) “Maximal green sequences for cluster-tilted algebras of finite type”[?] extends the results of [?] to cluster-tilted algebras of finite type and gives a conjectured formula for the maximum length of a MGS in these cases.
(c) (with PJ Apruzzese) “Stability conditions for affine type $A$”[?] uses [?], [?] to find the maximum length of a MGS for $\tilde{A}_{a,b}$ and determine which are linear.

(5) Papers about $m$-maximal green sequences:

(a) (with Y. Zhou) “Tame hereditary algebras ...”[?] gives a short module-theoretic proof that tame acyclic quivers have only finitely many $m$-maximal green sequences extending the theorem of [?] to the $m$-cluster case.

(b) “$m$-noncrossing trees,”[?] gives the $m$-cluster version of “cobinary trees” [?] and introduces the mutation formula for $m$-clusters in terms of the extended exchange matrix with an additional row for “slope” which, in [?], is the actual slope of an edge in the “$m$-noncrossing tree”.

(c) “Enumerating $m$-clusters ...”[?] reinterprets Fomin and Reading [?] using an $m$-cluster version of signed exceptional sequences [?]. The present lecture notes on “horizontal and vertical fans” are extracted from an early version of [?].

(d) “Horizontal and vertical mutation fans” constructs semi-invariant pictures to visualize $m$-maximal green sequences.

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