PROBLEMS SET 2 // DUE MONDAY 25

1.– Solve the congreunce $4x \equiv 10 \pmod{22}$

2.– Let k be a field. If $P(X) = \sum_{k=0}^{n} a_k x^k$, then we define $P'(x)$ as $\sum_{i=0}^{n-1}(i+1)a_{i+i}x^i$. We let D be the application from $k[X]$ to iteself which sends a polynomial $P(X)$ to $P'(X)$.

a.– Show that D is a group homomorphsim for the additive group structure of $k[X]$. Is it a ring homomorphism ?

b.– Let A be the set of $P(X) \in k[X]$ such that $P'(0) = 0$. Show that A is a subring of $k[X]$. Show that its fraction field is $k(X)$.

3.– Let R be a domain. Show that a unit in $R[X]$ is a constant polynomial c, with c a unit in R.

4.– Let $R = \mathbb{Z}_4$. Find a polynomial of degree 1 in $R[x]$ which is a unit.

5.– Let R be a domain, and denote by $\mathcal{F}(R, R)$ the ring of functions (or applications) from R to R. (It is define as we defined $\mathcal{F}(\mathbb{R}, \mathbb{R})$ in class : addition and multiplication of functions are defined values by values). We define a map $\psi : R[X] \to \mathcal{F}(R, R)$ by sending a polynomial $P(X)$ to the function form R to R which sends any element x of R to $P(x)$.

a.– Show that ψ is a ring homomorphism.

b.– Recall the example seen in class for $R = \mathbb{Z}_p$ which shows that ψ is not injective in this case.

c.– Assume that R is infinite. Show that ψ is injective.

d.– An element in the image of ψ is called a *polynomial function*. Show that the function sin in $\mathcal{F}(\mathbb{R}, \mathbb{R})$ is not a polynomial function. (Hence ψ is not surjective for $R = \mathbb{R}$.

e.– Assume $R = \mathbb{Z}_2$. Is ψ surjective ?