

PROBLEMS SET 5
DUE WEDNESDAY 16

1.– Let $P(X)$ be a polynomial of degree n in $C[X]$. Show that P has exactly n distinct roots in \mathbb{C} if and only if all its roots have multiplicity 1.

2.– Prove that $\mathbb{Q}(\sqrt{3}, \sqrt{7}) = \mathbb{Q}(\sqrt{3} + \sqrt{7})$.

3.– Let F be a finite field of cardinality $q = p^n$ with p prime and $n \geq 1$. Show (you can simply quote a theorem seen in class) that the multiplicative group F^* is cyclic. How many squares are there in F ?

4.– Let F be a finite field of cardinality $q = p^n$ with p prime and $n \geq 1$. Show that the application $x \mapsto x^p$ from F to F is a bijection.