EIGENVARIETIES, FAMILIES OF GALOIS REPRESENTATIONS, p-ADIC L-FUNCTIONS

MATH 201A, COURSE AT BRANDEIS, FALL 2010

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Prerequisites:

(i) Solid knowledge of commutative algebra and elementary algebraic geometry.
(ii) A good knowledge of the theory of modular forms: various definitions and their equivalence, Hecke operators, Galois representations, etc.
(iii) Some familiarity with Galois representations (complex, p-adic, and over finite field). If possible some p-adic Hodge theory (at least the notion of Hodge-Tate representations)
(iv) Basics of rigid analytic geometry.

Program:

1. The eigen-construction (1 week)

This will be a short and easy section discussing in details the simple and ubiquitous construction attaching to a commuting family of operators acting on a space or module a geometric object parameterizing its systems of eigenvalues.

2. Construction of the eigencurve à la Stevens (2 weeks)

In this section, we will explain in detail a construction of the eigencurve suggested by Stevens, using families of overconvergent modular symbols, We will also explain how to compare the product of this construction with the classical Coleman-Mazur's eigencurve.

3. p-adic L-functions of modular forms (1 week)

We will explain how to attach p-adic L-function to modular forms (and why it is important to do so), and how those L-functions are part of two-variables p-adic L-functions defined on the eigencurve.

4. Automorphic forms and higher eigenvarieties (2 weeks)

We will cover some construction of the eigenvarieties for more general reductive group than GL_2, at least Chenevier's construction in the case of forms of GL_n that are compact at infinity, and more (Emerton's, Ash-Stevens, etc.) if time allows.

5. Some p-adic Hodge-Tate theory (2 weeks)

To prepare the next section on families of Galois representations, we will cover some part (especially (ϕ, Γ)-modules) of the theory of p-adic Galois representations of a p-adic Galois group, alone and in families.

6. Refined families of Galois representations (2 weeks)

We will explain the notion of a refined family of Galois representations, and show that the family carried by the eigenvarieties are refined. We will show how to study the reducibility loci on such a family.

7. Geometry of eigenvarieties and applications (2 weeks)