The first part of this talk is an introduction to two topics: 1) the geometry and the topology of the Kodaira-Thurston manifold, 2) a geometric version of Fourier transform known as T-duality. In the second part of the talk we explore the geometry that arises when T-duality is applied to the Kodaira-Thurston manifold. In particular, we discuss some results and conjectures developed in collaboration with R. Heluani and O. Ben-Bassat.