Brandeis University

Everytopic Seminar

Friday, September 11
in room 226 at 1:40pm

Hilbert-Kunz multiplicities, rational, irrational algebraic(?), and transcendental(?)

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Abstract: Let A be a polynomial ring in $r+1$ variables over a field F of characteristic p. If f, not zero, is in A and q is a power of p, let I be the ideal of A generated by f and the q th powers of the variables. The Hilbert-Kunz multiplicity, μ, of f is the limit as q grows of $q^{-r}\dim(A/I)$. (The limit exists!).

In all cases where μ has been provably computed it is rational. Rationality holds for example when f is homogeneous in 3 variables or when F is finite and f is a sum of 2 variable polynomials in distinct variables. In my talk I'll discuss a conjecture about nodal cubics when $p=2$. If the conjecture holds I can show that when $p=2$:

(a) If F is finite and $f = x^3 + y^3 + xyz + h(u,v)$, then μ is algebraic, but not necessarily rational.

(b) There are 9 variables f with transcendental μ.

A result of Schneider about special values of hypergeometric functions is used to prove (b). It only remains to prove my conjecture!